

**TARGET DETECTION USING FRACTAL GEOMETRY**

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## ABSTRACT

The concepts and theory of Fractal Geometry were applied to the problem of segmenting a 256 x 256 pixel image so that manmade objects could be extracted from natural backgrounds. The two most important measurements necessary to extract these manmade objects were fractal dimension and lacunarity. Provision was made to pass the manmade portion to a lookup table for subsequent identification.

A computer program was written to construct cloud backgrounds of fractal dimensions which were allowed to vary between 2.2 and 2.8. Images of 3 model space targets were combined with these backgrounds to provide a data set for testing the validity of the approach.

Once the data set was constructed, computer programs were written to extract estimates of the fractal dimension and lacunarity on 4 x 4 pixel subsets of the image. It was shown that for clouds of fractal dimension 2.7 or less, appropriate thresholding on fractal dimension and lacunarity yielded a 64 x 64 edge-detected image with all or most of the cloud background removed. These images were enhanced by an erosion and a dilation to provide the final image passed to the lookup table.

While the ultimate goal was to pass the final image to a neural network for identification, this work shows the applicability of fractal geometry to the problems of image segmentation, edge detection and separating a target of interest from a natural background.

## INTRODUCTION AND MOTIVATION

Much of the work in automatic imaging of space scenes has been done assuming a dark sky background. This scenario, however, is the exception rather than the rule. Quite often a target will have the earth as its background thus providing a much more difficult problem for automatic image segmentation.

It has been observed that natural scenes are usually fractal in nature and have distinctly different fractal characteristics from those of manmade objects. Thus it should be possible to use Fractal Analysis as a way to separate manmade objects from a natural background. In this paper, we explore ways of doing this separation in a timely and efficient manner.

## NECESSARY CONCEPTS FROM FRACTAL GEOMETRY

Fractal Dimension - Fractal Geometry is a branch of mathematics developed by Benoit Mandelbrot and detailed in the book The Fractal Geometry of Nature [3]. In this work, Mandelbrot defines a new measure of the dimension of a set called the fractal or similarity dimension. This dimension is based on the notions of similarity and self-similarity.

A set A is said to be self-similar if it can be divided into N disjoint subsets each of which are exact replicas of the original set. A is said to be statistically self-similar if it can be divided into N disjoint subsets which are like - but not necessarily identical to - the original set. The fractal or similarity dimension of A is then determined from the following relation

$$D = \log(N)/\log(1/r) \text{ or } N = 1/r^D [3]$$

where D is the fractal dimension, N is the number of identical subsets into which A has been divided, and r is the ratio of the common size of the subsets to the original set A.

If A is Euclidean n-space, then D is an integer - in fact  $D = n$ . If D is not an integer, the set A is said to be a fractal.

The existence of a set with a non integer fractal dimension is illustrated in Figure 1. The figure illustrates the recursive rule for generating the Koch snowflake discussed in [5] and [7] among other places. At each iteration, straight line segments - such as the ones on the left - are replaced by the curve on the right. This procedure is allowed to continue forever. One can calculate that

$$\begin{aligned} N &= \text{number of replicas of the original set} = 4 \\ r &= \text{size of the replicas in relation to the original} = 1/3 \end{aligned}$$

$$\text{so that } D = \log(4)/\log(3) = 1.26....$$

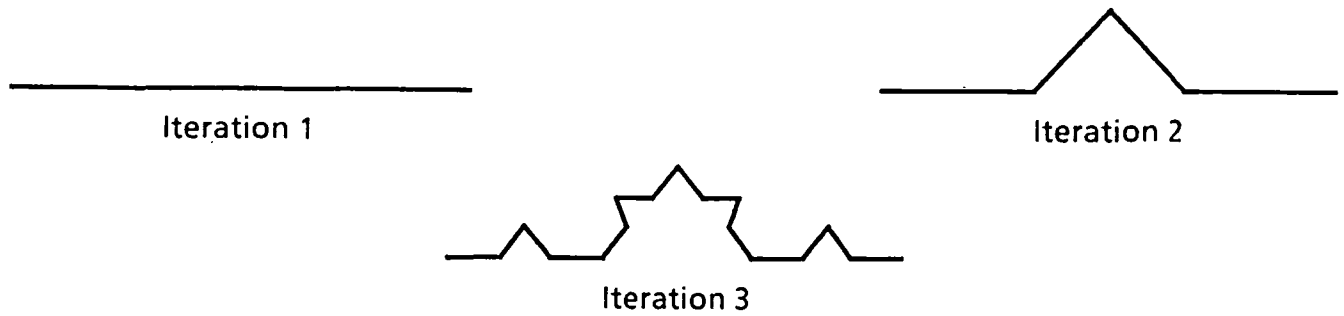


Figure 1

One can also calculate that the length of the curve generated by this process is infinite. Clearly the length of the curve at the  $k$ th iteration is  $4/3$  times its length at the  $(k-1)$ st iteration. Thus at iteration  $k$ , we have the length,  $l$ , given by

$l = l_0(4/3)^k$  where  $l_0$  is the length of the original segment.

The last calculation shows the relationship between the length of a fractal curve and the measuring device used to measure it. If the device is only capable of an accuracy of  $l_0$ , we would obtain a length of  $l_0$  when we measured the curve. If, however, the device had an accuracy of  $l_0(1/3)^k$ , we would obtain a length of  $l_0(4/3)^k$ . Thus as the measuring device becomes more accurate, the length of the curve increases without bound.

**Box Dimension:** The example above shows how to calculate the fractal dimension for an artificially generated fractal set. In nature - as well as in some artificially generated fractal sets - the calculation can be far more difficult. This may be due to the complexity of the set or due to the fact that the set has the property of statistical self similarity rather than self similarity.

A useful approximation to the fractal dimension is the box dimension described by Voss in [7]. To obtain the box dimension, the set is covered with a grid of boxes of size  $r$ . Let  $N(r)$  denote the number of boxes containing a point of the image. Then  $N(r)$  is proportional to  $1/r^D$ . By the appropriate choice of  $r$ , Voss shows that one obtains  $N(r) = 1/r^D$ .

**Lacunarity:** Mandelbrot showed [3] that the fractal dimension of a set alone was not always sufficient to characterize the appearance of the set. Sets with the same fractal dimension were shown to have different appearances due to texture or graininess. To differentiate between such sets, Mandelbrot introduced the concept of lacunarity which is defined to be

$$L = E((m/E(m)-1)^2) \quad [1, 3]$$

where  $E$  denotes expected value and  $m$  denotes the mass of the fractal set with some given density function  $P(m)$ . If the mass is close to the expected value - as in a compact, dense set - this quantity will be small. If the mass is distributed very unevenly this quantity will become larger. Thus lacunarity provides another way of characterizing sets.

Voss [7] provides a way of estimating lacunarity from an image by approximating the mass density function from the image data. Keller [1] provides an alternative yet similar measure of lacunarity. Each of these estimates may be calculated from information contained in the gray scale of the data.

## FRACTALS IN IMAGE PROCESSING

In work by Pentland [4], it is demonstrated that a scene may be segmented and classified by computing the fractal dimension of subsets of the image. His approach is based on the Fractal Imaging Theorem which describes the precise relationship between a fractal and the gray scale image of that fractal.

The author used a least squares fit to the Fourier power spectrum over  $8 \times 8$  pixel grids. By choosing appropriate break points in the histogram of the calculated fractal dimensions, Pentland was able to segment natural scenes into their components - e.g., water and land, water land and sky, etc.

In work by Keller et. al. [1, 2], image segmentation was achieved by using the fractal dimension - obtained from the interpolated box dimension developed in the paper - and the lacunarity feature described in the previous section. In these works, regions of like fractal dimension and lacunarity were determined by the k-means clustering algorithm. Segmentation was accomplished by plotting areas with similar characteristics in the same color.

## EDGE DETECTION USING FRACTAL GEOMETRY

Using the theory and research described in the previous two sections, work was begun to develop techniques which would permit the segmentation of an image so that manmade objects could be extracted from natural backgrounds. In addition, it was desired that the segmentation be performed more rapidly than is possible with the techniques previously described. The work by Pentland in [4] required the calculation of FFT's, a regression step, and the construction of a histogram of fractal dimensions. Keller's approach in [1] required several passes over the image and the application of a clustering technique before the segmentation was performed.

It was not expected - nor did it turn out - that the overall segmentation achieved by a faster approach would be as good as that achieved in the previous techniques. In fact, the only segmentation desired was a division of the scene into two parts - the part containing the manmade space object and the part containing everything else.

Approach: An existing computer program was used to generate cloud cover which had fractal dimensions of 2.2, 2.4, 2.5, 2.6, 2.7, and 2.8. Images of model toy space shuttles and a model satellite - representing manmade objects - were captured and combined with the clouds to form  $256 \times 256$  pixel images with 16 colors or gray scale steps.

Once the images were created, programs were written to extract the box dimension and lacunarity measurements on grids of size  $4 \times 4$ ,  $8 \times 8$ , and  $16 \times 16$ .

Simple thresholding was then performed on the fractal dimensions and lacunarity measurements taken. Thus the program was written to allow the user to input threshold values and display only those grids whose fractal dimensions, exceeded (fell below) the fractal threshold and whose lacunarity measurement exceeded (fell below) the lacunarity threshold. In addition, an  $m \times m$  grid was displayed (where  $m = 256/n$  and  $n$  is 4, 8, or 16). A pixel in the  $m \times m$  grid would be "on" if and only if the corresponding  $n \times n$  grid passed the fractal and lacunarity threshold tests. It was this final  $m \times m$  grid which was sent to a lookup table and ultimately will be sent to a neural network or other pattern recognizer.

**Implementation and Improvements:** The computer programs described above were written and applied to the data sets. While it was not the case that it was always possible to recover the manmade object from the background completely, it was observed that this approach generally does an excellent job of edge detection on the manmade objects.

It was determined - experimentally - that the  $4 \times 4$  grids worked better than the  $8 \times 8$  or  $16 \times 16$  grids. While the latter two grid sizes performed a good job of edge detecting the manmade objects, they also tended to select boundary areas in the clouds. In particular, it was difficult to find appropriate thresholds which would remove those regions where clouds met regions of clear sky (or open ocean depending on the orientation of the observer to the target).

Using  $4 \times 4$  grids was not without difficulty. The major problem was that when computing the box dimension based on the gray scale of the image, the algorithm described in [7] requires that the box be scaled by the same amount in each of the  $x$ ,  $y$ , and  $z$  directions. This allowed for the choice of  $r$  to be 1,  $1/2$ , or  $1/4$ . If  $r = 1$  nothing is gained since there is one box. If  $r = 1/4$ ,  $N(r)$  must necessarily be 4 (one box per pixel) and we would obtain  $D = 1$ . Thus the only choice was to select  $r = 1/2$ . This scale factor, however, allowed for little or no separation in the  $z$  or gray scale direction. With 2 vertical boxes, gray scales 0-7 were grouped together and those from 8-15 were grouped together.

To overcome this problem, it was decided to scale by  $1/2$  in each of the  $x$  and  $y$  directions and by  $1/8$  in the  $z$  direction. While this gave a distorted fractal dimension when the formula for computing the box dimension was applied, it allowed for better detection of variations in the gray scale. The distorted measurements were converted to more realistic, believable fractal dimensions by using a linear scaling to make them all lie in the interval  $\{2, 3\}$ . This conversion is not necessary from a mathematical point of view, but rather was done for aesthetic reasons.

For the higher fractal dimensions of clouds, it was noticed that occasional stray, isolated pixels were turned on in the  $64 \times 64$  image to be passed into the lookup table. In hopes of achieving a more distortion free image, an erosion step was added to remove stray pixels and a dilation step was inserted to strengthen the detected edges. In the erosion step, any on pixel with no neighbors was turned off. For the dilation step, we used a  $2 \times 2$  square of pixels and performed the dilation as described in [6].

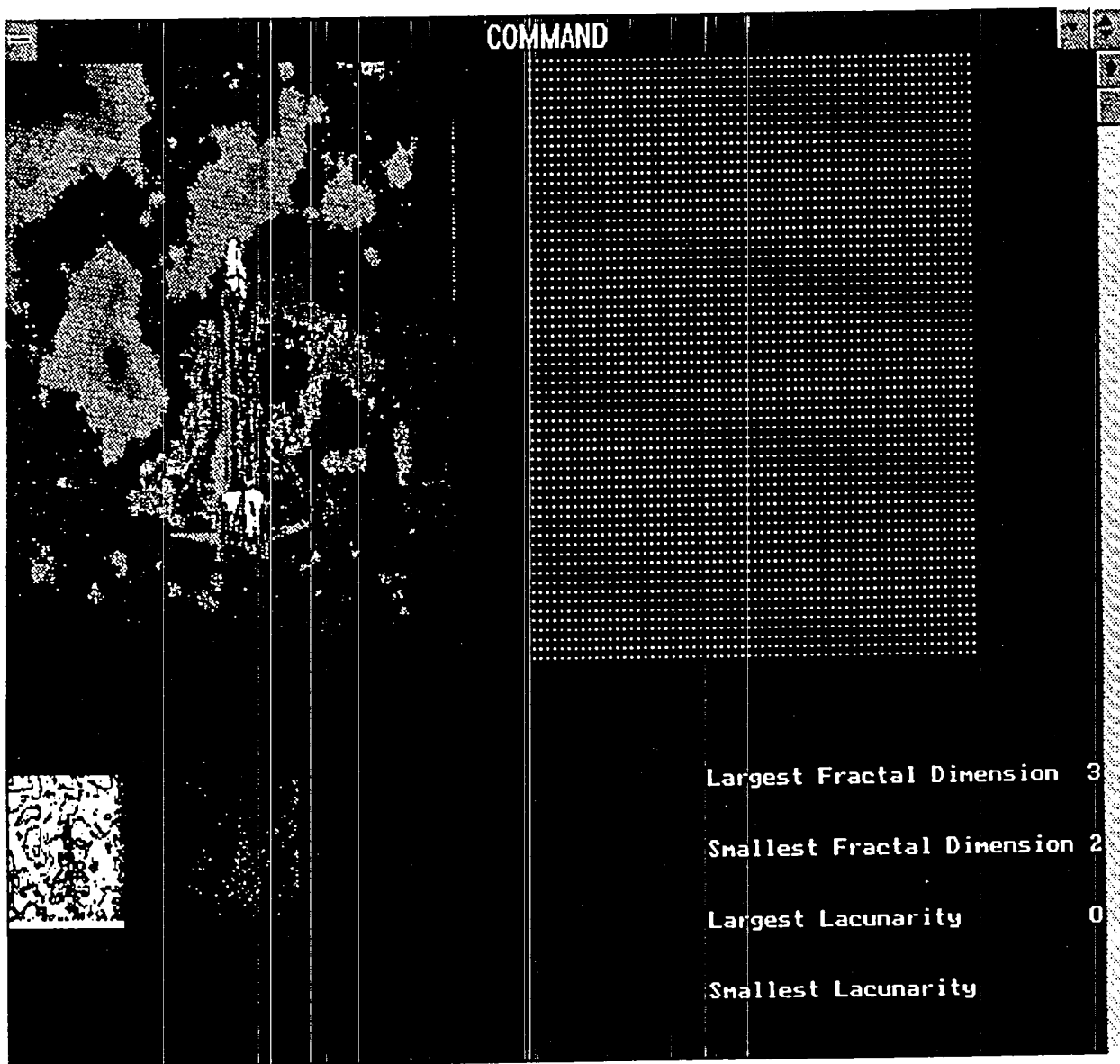


FIGURE 2

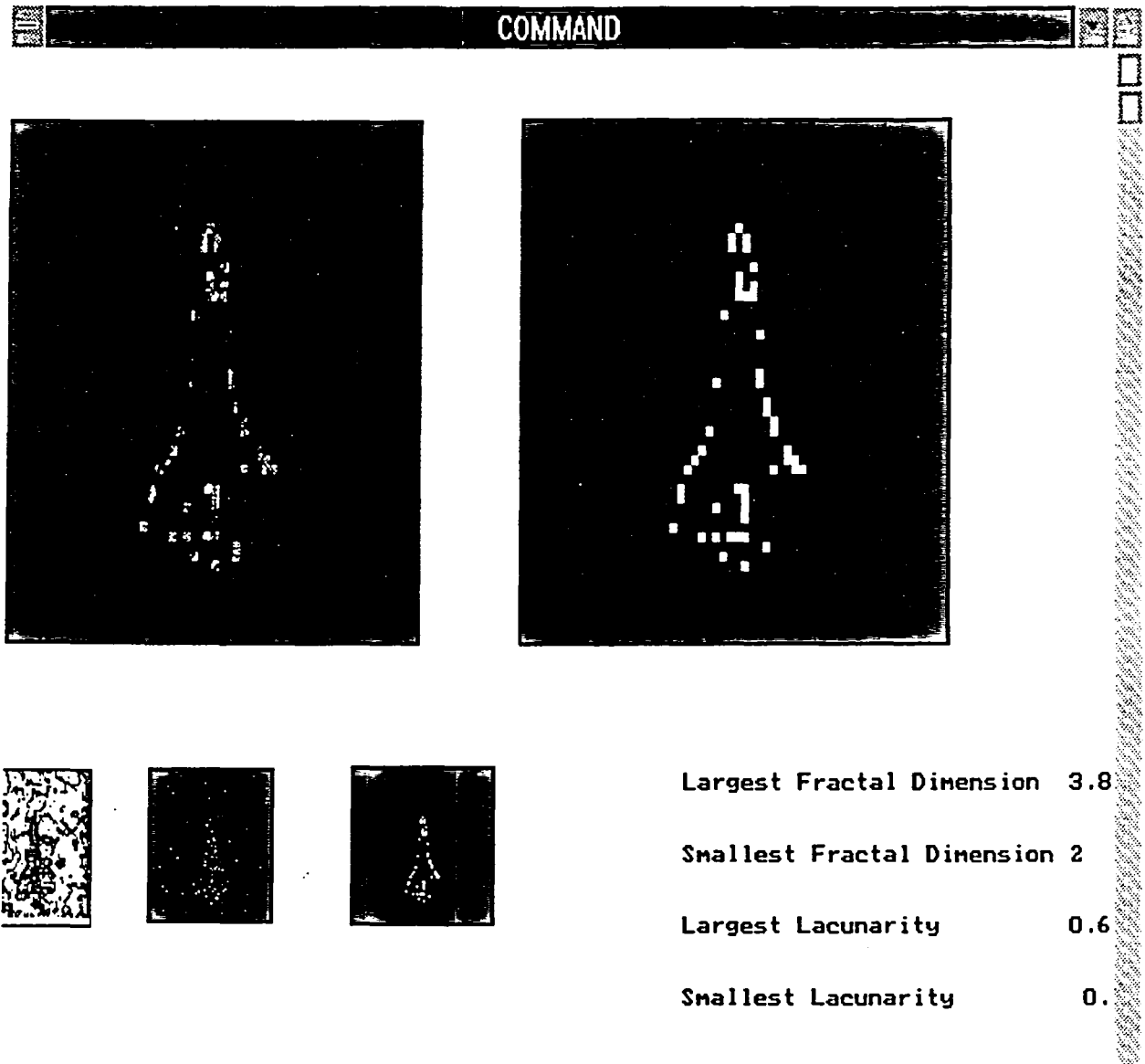


FIGURE 3

## SUGGESTIONS FOR FURTHER STUDY

Most of the emphasis of this work was placed on the development of the target detection portion of the program. The participants in the work hope to carry the work further and to replace the table lookup described earlier with an appropriate pattern identifier, e.g., neural network. This would allow automatic identification as well as automatic detection of space targets. In addition, the authors believe that other image processing techniques - e.g. openings and closings - may enhance the quality of the 64 x 64 image being passed to the pattern recognizer.

Finally, the technique must be verified on real data. While the model targets and artificially generated backgrounds go far to proving the validity of the concept, it should be verified on actual images from space. Furthermore, the proof of concept has been restricted to the open ocean, the open sky, or backgrounds representing a combination of clouds, ocean, and sky. However no provision was made to test it against a background containing land masses e.g. continents and/or islands.

Cooperation between the authors is expected to continue in the future on the solution of these problems.

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